DSC-Vol. 57-2, 1995 IMECE
Proceedings of the ASME Dynamic Systems and Control Division
ASME 1995

A LAGRANGIAN QUASICOORDINATE FORMULATION FOR DYNAMIC SIMULATIONS OF MULTIBODY SYSTEMS

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ABSTRACT

Utilizing the work of Quinn (1990), a new extension of Lagrange's equation in terms of quasicoordinates is presented. "Global" forms of velocity and kinetic energy have been used in conjunction with identities introduced by Quinn, allowing a single equation, in a form similar to Newton's 2nd law, to represent the entire set of rotational equations of motion for a system. This method may be used if the energies are explicit functions of angular velocities and coordinate transformation matrices. A new identity is introduced which represents the elimination of Coriolis terms for the entire system. The formulation is applicable to a large class of problems in the dynamics of structures, including spacecraft, robots, ground vehicles and aircraft.

This formulation has been used in the simulation of a nontrivial problem. Roboticists and biologists, working with biologically inspired robotics, are interested in the exceptional locomotion capabilities of the cockroach. A 36 degree of freedom model is used to represent the movement of the cockroach, *Blaberus discoidalis*. Six 5 degree of freedom legs support a freely translating and rotating body. The model is driven by joint angles obtained by processing high speed video of the cockroach's locomotion, and mass and length parameters are taken directly from the roach. Results have been consistent with those obtained previously by other researchers.

INTRODUCTION

Legged locomotion may permit the negotiation of very rough terrain. Insects, for example, show great skill and robustness in this task. Hence, there is an interest in building walking robots with mechanical designs and locomotion controllers inspired by those of insects. Many insect-based robots have been developed recently which show promise for walking autonomously on rough terrain (Donner, 1987; Brooks, 1989; Ferrell, 1993; Pfeiffer et al., 1994; Espenschied et al., 1994).

Based on this success of using biological inspiration to build walking machines, it is clear that a better understanding of the insect's mechanics and control system is necessary to take further advantage of this strategy. To help gain this understanding, it is useful to construct a dynamic model of the insect so that simulations may be performed along with biological experiments.

A dynamic simulation of the walking-stick insect was developed to aid in the design of a hexapod robot which is kinematically similar to this insect (Pfeiffer et al., 1991; Pfeiffer et al., 1994). An optimization strategy was used to determine the joint torques in the simulation because there are more unknowns (joint torques and ground reactions) in the formulation than equations. A simplified simulation of a robot which is loosely based on the stick insect has also been developed (Quinn and Lin, 1993). In this case, the mass of the legs which are in stance is neglected, permitting a closed-form relationship between the ground reactions and the joint torques. Also, proportional-derivative feedback control was used to drive the joints to follow their given moving equilibrium points according to the hypothesized mammalian muscle model (Bizzi and Mussa-Ivaldi, 1990). A dynamic simulation has been incorporated into a three dimensional animation of animal and robot running (Raibert and Hodgins, 1991).

Lagrange's equation in terms of quasicoordinates is especially useful for formulating the equations of motion (EOMs) for structures which undergo finite rotations in three dimensions (Meirovitch, 1970). In this paper, a formulation of Lagrange's equation in terms of quasicoordinates developed previously (Quinn, 1990) has been extended into a global form for tree-like multibody structures. A "global" form of kinetic energy is used in conjunction with identities introduced in the previous paper, permitting a single matrix equation, in a form similar to Newton's 2nd law, to represent the entire set of rotational equations of motion for a system. This method may be used if the energies are explicit functions of angular velocities and coordinate transformation matrices, which is the case for many multibody systems. This formulation is applicable to a large class of problems in the dynamics of structures, including spacecraft, robots, ground vehicles, and aircraft.

One of the drawbacks of a Lagrangian formulation is that it results in many Coriolis terms which, when added, are zero. The elimination of these terms results in an efficient formulation. However, in a general matrix approach, these terms are difficult to identify. In this paper a global form of an identity is introduced which permits a straightforward cancellation of these terms.

As an example, this new dynamic formulation for multibody systems is applied to the simulation of a cockroach (*Blaberus discoidalis*) for the purpose of designing a walking robot which is kinematically similar to the insect. A 36 degree of freedom (DOF) model is used to represent the movement of the animal. There are six legs and each leg has three rigid segments (coxa, femur, and tibia) and five rotational degrees of freedom: three at the body-coxa joint, one at the coxa-femur joint, and one at the femur-tibia joint. The animal's body is treated as rigid and it is free to translate and rotate in three dimensions. The insect's mechanics are actually more complex, including a flexible fourth leg segment called the tarsus and additional body segments.

The input to the simulation includes the massive, inertial, and geometric properties of the animal. The joint motions of the animal as functions of time are also input to the simulation. This data is determined by digitizing and processing high-speed video of the cockroach. The output of the simulation includes joint torques, body motions, and ground reactions. Body motions can be compared directly with those of the insect on videotape. Ground reactions have been measured directly by biologists (Full et al., 1991). Preliminary results have been consistent with those obtained previously (Quinn and Lin, 1993; Full et al., 1991).

DYNAMIC FORMULATION

Lagrange's Equation in Terms of Quasicoordinates

Considering only structures undergoing finite rotations in three dimensional space, one form (Quinn, 1990) of Lagrange's equation in terms of absolute quasicoordinates ($\underset{\sim}{\omega}$) can be expressed as

$$\frac{d}{dt}\left(\frac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_B}\right) + \tilde{\omega}_B \frac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_B} - \frac{\partial \overline{T}}{\partial C_{NB}} + \frac{\partial \overline{V}}{\partial C_{NB}} = \underset{\sim}{M}, \quad (1)$$

where the following definitions are used. $\underset{\sim}{M}$ is a set of moments which are defined by virtual work requirements. C is an orthogonal rotational transformation matrix from a body-fixed reference frame to an inertial reference frame (N-frame). $\underset{\sim}{\omega}$ is the absolute angular velocity of a given body, expressed with respect to its own reference frame. $\tilde{\omega}$ is a skew symmetric matrix operation on the vector $\underset{\sim}{\omega}$, defined as

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \quad (2)$$

$\overline{T} = \overline{T}(\underset{\sim}{\omega}, C)$ and $V = V(C)$ are the kinetic and potential energies, respectively. When the energy terms of $\overline{T}$ or $\overline{V}$ take the following form,

$$\overline{T} = \underset{\sim}{x}^T C_{NB} \underset{\sim}{y}, \quad (3)$$

then the notation below helps to make Eq.(1) a straightforward and more efficient way of formulating EOMs for complex structures:

$$\frac{\partial}{\partial C_{NB}}(\overline{T}) = \tilde{y} C_{BN} \underset{\sim}{x}. \quad (4)$$

Indeed, energy terms in both $\overline{T}$ and $\overline{V}$ can be algebraically manipulated to take the form of Eq.(3).

Several steps are involved in using Eq.(1). The foremost is to develop expressions for the position, $\underset{\sim}{p}$, of differential masses in each body of the system with respect to an inertial reference N-frame. For a body i in a group of M bodies,

$$\underset{\sim}{p}_i = \underset{\sim}{R}_1 + C_{N1} \underset{\sim}{L}_1 + C_{N2} \underset{\sim}{L}_2 + \cdots + C_{Ni} \underset{\sim}{\rho}_i, \quad (5)$$

where $\underset{\sim}{R}_1$ is the position of the origin of the body fixed reference frame of body 1, expressed with respect to the N-frame. $\underset{\sim}{L}_i$ is the position of the (i+1)-frame origin in the i-frame, expressed with respect to the i-frame. $\underset{\sim}{\rho}_i$ is the position of a differential mass in the i-frame, expressed with respect to the i-frame.

Differentiation of Eq.(5) produces the following:

$$\dot{\underset{\sim}{p}}_i = \dot{\underset{\sim}{R}}_1 + C_{N1}\tilde{\omega}_1 \underset{\sim}{L}_1 + C_{N2}\tilde{\omega}_2 \underset{\sim}{L}_2 + \cdots + C_{Ni}\tilde{\omega}_i \underset{\sim}{\rho}_i. \quad (6)$$

Here we limit our discussion to rigid-body systems, and systems in which $\overline{T} = \overline{T}(\underset{\sim}{\omega}, C)$ and $\overline{V} = \overline{V}(C)$. This would encompass a large class of problems, specifically tree-like multibody systems, such as aircraft, spacecraft, robots, ground vehicles, legs and arms, or with some discretization, snake-like structures.

It then becomes necessary to perform an inner product on each velocity expression, which is then inserted into an expression for kinetic energy. The total kinetic energy, $\overline{T}$, becomes

$$\overline{T} = \sum_{i=1}^{M} T_i = \sum_{i=1}^{M}\left[\frac{1}{2}\int \dot{\underset{\sim}{p}}_i^T \dot{\underset{\sim}{p}}_i \, dm_i\right], \quad (7)$$

where M is the number of bodies. Considering Eqs.(6) and (7), it becomes apparent that a sizeable amount of algebra will be required to perform the necessary operations contained in Eq.(1). To aid in this simplification step, the following helpful identity was introduced, although the algebra required for large problems remains tedious. Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be arbitrary (3x1) vectors, then

$$\tilde{a}\tilde{b} + \tilde{b}^T \tilde{a} + [\widetilde{\tilde{b}\underset{\sim}{a}}] = 0. \quad (8)$$

$\overline{T}$ is also substituted into Eq.(1) for each body for which the EOMs are desired.

A Generalized Approach for Open-Chain Multibody Systems

Because we are working with a multibody system, it is advantageous to express the velocity of the system in a "global" matrix form, such as

$$\begin{bmatrix} \dot{\underset{\sim}{p}}_1 \\ \dot{\underset{\sim}{p}}_2 \\ \vdots \\ \dot{\underset{\sim}{p}}_M \end{bmatrix} = \begin{bmatrix} \dot{\underset{\sim}{R}}_1 \\ \dot{\underset{\sim}{R}}_1 \\ \vdots \\ \dot{\underset{\sim}{R}}_1 \end{bmatrix} + \begin{bmatrix} C_{N1}\tilde{\rho}_1^T & 0 & \cdots & 0 \\ C_{N1}\tilde{L}_1^T & C_{N2}\tilde{\rho}_2^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1}\tilde{L}_1^T & C_{N2}\tilde{L}_2^T & \cdots & C_{NM}\tilde{\rho}_M^T \end{bmatrix} \begin{bmatrix} \underset{\sim}{\omega}_1 \\ \underset{\sim}{\omega}_2 \\ \vdots \\ \underset{\sim}{\omega}_M \end{bmatrix}, \quad (9)$$

or in an abbreviated form

$$\dot{\underset{\sim}{p}}_G = \dot{\underset{\sim}{R}}_G + \Phi \underset{\sim}{\omega}_G . \quad (10)$$

We can also use another "global" form (subscript 'G') for Eq.(6), such as

$$\dot{\underset{\sim}{p}}_G = \dot{\underset{\sim}{R}}_G + \begin{bmatrix} C_{N1} & 0 & 0 & 0 & \cdots & 0 \\ 0 & C_{N1} & C_{N2} & 0 & \cdots & 0 \\ 0 & C_{N1} & 0 & C_{N2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & C_{N1} & 0 & C_{N2} & \cdots & C_{NM} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_1 \underset{\sim}{\rho}_1 \\ \tilde{\omega}_1 \underset{\sim}{L}_1 \\ \tilde{\omega}_2 \underset{\sim}{\rho}_2 \\ \tilde{\omega}_2 \underset{\sim}{L}_2 \\ \vdots \\ \tilde{\omega}_M \underset{\sim}{\rho}_M \end{bmatrix}, \quad (11)$$

or

$$\dot{\underset{\sim}{p}}_G = \dot{\underset{\sim}{R}}_G + \hat{C} \underset{\sim}{\Psi}_G . \quad (12)$$

Comparing Eqs.(10) and (12), it becomes immediately apparent that

$$\Phi \underset{\sim}{\omega}_G = \hat{C} \underset{\sim}{\Psi}_G . \quad (13)$$

Notice that the total kinetic energy for the system can now be expressed as

$$\overline{T} = \frac{1}{2} \int \dot{\underset{\sim}{p}}_G^T [d\overline{m}_G] \dot{\underset{\sim}{p}}_G , \quad (14)$$

where $[d\overline{m}_G]$ is diagonal:

$$[d\overline{m}_G] = \begin{bmatrix} [d\overline{m}_1] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [d\overline{m}_M] \end{bmatrix}, \quad [d\overline{m}_i] = \begin{bmatrix} dm_i & 0 & 0 \\ 0 & dm_i & 0 \\ 0 & 0 & dm_i \end{bmatrix}, \quad (15)$$

and dm_i is a differential mass in body i. At this point, let the following definitions apply:

$$\begin{bmatrix} \dfrac{d}{dt}\left(\dfrac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_1}\right) + \tilde{\omega}_1 \dfrac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_1} \\ \vdots \\ \dfrac{d}{dt}\left(\dfrac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_M}\right) + \tilde{\omega}_M \dfrac{\partial \overline{T}}{\partial \underset{\sim}{\omega}_M} \end{bmatrix} = \underset{\sim}{\eta}' , \quad (16)$$

$$\begin{bmatrix} \dfrac{\partial \overline{T}}{\partial C_{N1}} \\ \vdots \\ \dfrac{\partial \overline{T}}{\partial C_{NM}} \end{bmatrix} = \underset{\sim}{\eta}'' . \quad (17)$$

Substituting Eq.(12) into Eq.(14), and performing the manipulations described previously by Eqs.(3) and (4), we can show the following:

$$\underset{\sim}{\eta}'' = \int (Q \tilde{\Psi}_G \hat{C}^T [d\overline{m}_G] \dot{\underset{\sim}{R}}_G + Q \tilde{\Psi}_G \hat{C}^T [d\overline{m}_G] \hat{C} \underset{\sim}{\Psi}_G) , \quad (18)$$

where

$$Q = \begin{bmatrix} I & I & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I \end{bmatrix}, \quad (19)$$

$$\tilde{\Psi}_G = \begin{bmatrix} \widetilde{[\tilde{\omega}_1 \underset{\sim}{\rho}_1]} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \widetilde{[\tilde{\omega}_1 \underset{\sim}{L}_1]} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \widetilde{[\tilde{\omega}_2 \underset{\sim}{\rho}_2]} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \widetilde{[\tilde{\omega}_2 \underset{\sim}{L}_2]} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \widetilde{[\tilde{\omega}_M \underset{\sim}{\rho}_M]} \end{bmatrix}, \quad (20)$$

and 'I' is a (3x3) identity matrix. If Eq.(10) is substituted into Eq.(14), and then into Eq.(1), the following results:

$$\begin{aligned} \underset{\sim}{\eta}' = \int (& \dot{\Phi}^T [d\overline{m}_G] \dot{\underset{\sim}{R}}_G + \Phi^T [d\overline{m}_G] \ddot{\underset{\sim}{R}}_G + \dot{\Phi}^T [d\overline{m}_G] \Phi \underset{\sim}{\omega}_G + \\ & \Phi^T [d\overline{m}_G] \dot{\Phi} \underset{\sim}{\omega}_G + \Phi^T [d\overline{m}_G] \Phi \dot{\underset{\sim}{\omega}}_G + \\ & \tilde{\omega}_G \Phi^T [d\overline{m}_G] \dot{\underset{\sim}{R}}_G + \tilde{\omega}_G \Phi^T [d\overline{m}_G] \Phi \underset{\sim}{\omega}_G), \end{aligned} \quad (21)$$

where

$$\dot{\Phi} = \begin{bmatrix} C_{N1}\tilde{\omega}_1\tilde{\rho}_1^T & 0 & \cdots & 0 \\ C_{N1}\tilde{\omega}_1\tilde{L}_1^T & C_{N2}\tilde{\omega}_2\tilde{\rho}_2^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1}\tilde{\omega}_1\tilde{L}_1^T & C_{N2}\tilde{\omega}_2\tilde{L}_2^T & \cdots & C_{NM}\tilde{\omega}_M\tilde{\rho}_M^T \end{bmatrix}, \quad (22)$$

$$\tilde{\omega}_G = \begin{bmatrix} \tilde{\omega}_1 & 0 & \cdots & 0 \\ 0 & \tilde{\omega}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{\omega}_M \end{bmatrix}. \quad (23)$$

Subtracting Eq.(18) from Eq.(21), we get

$$\underset{\sim}{\eta}' - \underset{\sim}{\eta}'' = \int \Phi^T [d\overline{m}_G] \left(\ddot{\underset{\sim}{R}}_G + \dot{\Phi} \underset{\sim}{\omega}_G + \Phi \dot{\underset{\sim}{\omega}}_G \right) . \quad (24)$$

This is obtained by noting that

$$\dot{\Phi} + \Phi\tilde{\omega}_G^T + \hat{C}\tilde{\Psi}_G Q^T = 0 \,. \tag{25}$$

Eq.(25) represents a global form of Eq.(8), which permits a global elimination of Coriolis terms in the EOMs. More simplification reveals the basic structure of the formulation:

$$\underset{\sim}{\eta}' - \underset{\sim}{\eta}'' = \int \Phi^T [d\overline{m}_G] \frac{d}{dt}\left(\dot{\underset{\sim}{R}}_G + \Phi \underset{\sim}{\omega}_G\right) = \int \Phi^T [d\overline{m}_G] \frac{d}{dt}\left(\dot{\underset{\sim}{p}}_G\right) \tag{26}$$

If we consider the translational EOM for the system, represented by Lagrange's equation, we can use the previous global expressions to produce the following:

$$\frac{d}{dt}\left(\frac{\partial \overline{T}}{\partial \dot{\underset{\sim}{R}}_1}\right) - \frac{\partial \overline{T}}{\partial \underset{\sim}{R}_1} = \int I_V^T [d\overline{m}_G]\left(I_V \ddot{\underset{\sim}{R}}_1 + \dot{\Phi}\underset{\sim}{\omega}_G + \Phi \dot{\underset{\sim}{\omega}}_G\right), \tag{27}$$

where

$$I_V^T = [I \,\vdots\, I \,\vdots\, \cdots \,\vdots\, I]^T . \tag{28}$$

Therefore, we can conclude that the following represents the kinetic energy terms in the EOMs for the entire multibody system:

$$\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial \overline{T}}{\partial \dot{\underset{\sim}{R}}_1}\right) - \frac{\partial \overline{T}}{\partial \underset{\sim}{R}_1} \\ \cdots\cdots\cdots \\ \underset{\sim}{\eta}' - \underset{\sim}{\eta}'' \end{bmatrix} = \tag{29}$$

$$\int \left[\begin{array}{c:c} I_V^T [d\overline{m}_G] I_V & I_V^T [d\overline{m}_G] \Phi \\ \hdashline \Phi^T [d\overline{m}_G] I_V & \Phi^T [d\overline{m}_G] \Phi \end{array}\right] \begin{bmatrix} \ddot{\underset{\sim}{R}}_1 \\ \hdashline \dot{\underset{\sim}{\omega}}_G \end{bmatrix} + \left[\begin{array}{c} I_V^T [d\overline{m}_G] \dot{\Phi} \\ \hdashline \Phi^T [d\overline{m}_G] \dot{\Phi} \end{array}\right] \underset{\sim}{\omega}_G .$$

As expected, the inertia matrix is symmetric. The centrifugal terms are included in the second term on the right side. The top equation on the right side of Eq.(29) accounts for the translational terms, while the lower equation accounts for all of the rotational terms of the entire system. Potential energy terms are discussed below.

Integration takes place over the mass of the bodies of the system, and may not be intuitive. The following general expressions should apply in most cases:

$$\overline{C} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C_{N1} & 0 & \cdots & 0 & 0 \\ C_{N1} & C_{N2} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 \\ C_{N1} & C_{N2} & \cdots & C_{N(M-1)} & 0 \end{bmatrix}, \tag{30}$$

$$C^D = \begin{bmatrix} C_{N1} & 0 & \cdots & 0 \\ 0 & C_{N2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{NM} \end{bmatrix}, \tag{31}$$

$$\int \underset{\sim}{\rho}_i \, dm_i = \underset{\sim}{S}_i \, m_i \qquad \int \tilde{\rho}_i^T \tilde{\rho}_i dm_i = I_i , \tag{32}$$

$$\int I_V^T [d\overline{m}_G] \Phi = I_V^T \overline{m}_G \overline{\Phi} = I_V^T \overline{m}_G (\overline{C}\tilde{L}_G^T + C^D \tilde{S}_G^T) , \tag{33}$$

$$\int \Phi^T [d\overline{m}_G] \Phi = \overline{\Phi}^T \overline{m}_G \overline{\Phi} + I_G^* , \tag{34}$$

$$\int \Phi^T [d\overline{m}_G] \dot{\Phi} = \overline{\Phi}^T \overline{m}_G \dot{\overline{\Phi}} + \tilde{\omega}_G I_G^* = \overline{\Phi}^T \overline{m}_G (\overline{C}\tilde{\omega}_G \tilde{L}_G^T + C^D \tilde{\omega}_G \tilde{S}_G^T) + \tilde{\omega}_G I_G^* , \tag{35}$$

$$I_G^* = \begin{bmatrix} I_1 & 0 & \cdots & 0 \\ 0 & I_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_M \end{bmatrix} + \tilde{S}_G \tilde{S}_G \overline{m}_G = I_G + \tilde{S}_G \tilde{S}_G \overline{m}_G . \tag{36}$$

The Φ matrix can also be used when working with the potential energy terms. Considering only gravity, the potential energy can be expressed as

$$\overline{V} = -\int \underset{\sim}{g}_G^T [d\overline{m}_G] \underset{\sim}{p}_G = -\int \underset{\sim}{g}_G^T [d\overline{m}_G](I_V \underset{\sim}{R}_1 + \overline{C}\underset{\sim}{L}_G + C^D \underset{\sim}{\rho}_G) , \tag{37}$$

where $\underset{\sim}{g}_G = I_V \underset{\sim}{g}$. Therefore, it can be shown that

$$\frac{\partial \overline{V}}{\partial \underset{\sim}{R}_1} = -\int I_V^T [d\overline{m}_G] \underset{\sim}{g}_G = -I_V^T \overline{m}_G \underset{\sim}{g}_G , \tag{38}$$

$$\begin{bmatrix} \frac{\partial \overline{V}}{\partial C_{N1}} \\ \vdots \\ \frac{\partial \overline{V}}{\partial C_{NM}} \end{bmatrix} = -\int \Phi^T [d\overline{m}_G] \underset{\sim}{g}_G = -\overline{\Phi}^T \overline{m}_G \underset{\sim}{g}_G . \tag{39}$$

Transforming the EOMs to True Coordinates

In general, the rotational EOMs can be expressed in terms of three different types of coordinates, those being absolute quasi-coordinates ($\underset{\sim}{\omega}$) with applied moments $\underset{\sim}{M}_G$, relative quasicoordinates ($\underset{\sim}{\Omega}_G$) with applied moments $\underset{\sim}{M}_{rel,G}$, and true coordinates ($\underset{\sim}{\alpha}_G$) with applied moments $\underset{\sim}{\tau}_G$ (e.g. torques due to joint motors and actuators). The following relationship exists between $\underset{\sim}{\omega}_G$ and $\underset{\sim}{\Omega}_G$:

$$\underset{\sim}{\omega}_G = C^R \underset{\sim}{\Omega}_G , \tag{40}$$

where

$$C^R = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ C_{21} & I & \cdots & 0 & 0 \\ C_{31} & C_{32} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & I & 0 \\ C_{M1} & C_{M2} & \cdots & C_{M(M-1)} & I \end{bmatrix} . \tag{41}$$

Virtual work then provides the following relationship:

$$\underset{\sim}{M}_{rel,G} = C^{RT} \underset{\sim}{M}_G . \tag{42}$$

A link to true coordinates is available as follows:

$$\underset{\sim}{\Omega}_G = D \dot{\underset{\sim}{\alpha}}_G , \tag{43}$$

and

$$D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_M \end{bmatrix}_{j \times k}, \quad \text{where} \begin{cases} j = 3M \\ k = \#\text{ rotational DOF} \end{cases} . \tag{44}$$

Virtual work leads to the following relationship:

$$\tau_G = D^T M_{rel,G} . \quad (45)$$

Thus, Eqs.(42) and (45) provide a means by which the absolute quasicoordinate EOMs can be transformed into true coordinate EOMs:

(true coordinate EOMs) = D^T(relative quasicoordinate EOMs)
= $D^T C^{RT}$(absolute quasicoordinate EOMs). (46)

The matrices D and $\dot{D}$ are formulated based upon the system to be modeled, and are nonlinear functions of α_G and $\dot{\alpha}_G$. This process is demonstrated below for a cockroach leg. We now have everything needed to present a final form for the EOMs in terms of true coordinates:

$$\overline{D} = \begin{bmatrix} I & 0 \\ 0 & D \end{bmatrix}, \qquad \overline{C}^R = \begin{bmatrix} I & 0 \\ 0 & C^R \end{bmatrix}, \quad (47)$$

$$\left[\overline{D}^T\overline{C}^{RT}\begin{bmatrix} I_v^T\overline{m}_G I_v & I_v^T\overline{m}_G\overline{\Phi} \\ \overline{\Phi}^T\overline{m}_G I_v & \overline{\Phi}^T\overline{m}_G\overline{\Phi}+I_G^* \end{bmatrix}\overline{C}^R\overline{D}\right]\begin{bmatrix}\ddot{R}_I \\ \ddot{\alpha}_G\end{bmatrix} = \begin{bmatrix}F^* \\ \tau^*\end{bmatrix}, \quad (48)$$

where

$$\begin{bmatrix}F^* \\ \tau^*\end{bmatrix} = -\overline{D}^T\overline{C}^{RT}\begin{bmatrix} I_v^T\overline{m}_G\Gamma \\ \overline{\Phi}^T\overline{m}_G\Gamma+\tilde{\omega}_G I_G^*\omega_G+I_G^*C^R\xi \end{bmatrix} + \begin{bmatrix}F \\ \tau_G\end{bmatrix}, \quad (49)$$

and

$$\Gamma = \dot{\overline{\Phi}}\omega_G + \overline{\Phi}C^R\xi - g_G, \qquad \xi = \tilde{\omega}_G\Omega_G + \dot{D}\dot{\alpha}_G . \quad (50)$$

Substitution can be made from Eqs.(40) and (43), such that the EOMs will be entirely in terms of true coordinates. Eq.(48) is presented as a general EOM for any multi-rigid-body system, acted upon by gravity, that meets the following requirement: the global velocity of the system can be expressed as shown by Eq.(9).

Example: An Algorithm for Formulating EOMs of a Cockroach

Consider Figures 1 and 2, which show two configurations of a system consisting of a modeled cockroach leg attached to a parallelepiped. Figure 1 depicts the system in its "zeroed" state, that being when all joint angles are defined as zero. Figure 2 shows the leg and body in a general position, which is in a region of normal locomotion in configuration space. The sense of the joint angles can be seen in Figure 2. There are a total of eleven DOFs which are used to model this system.

The parallelepiped is used here to represent the main body of the cockroach, which is allowed to translate and rotate freely. The modeled leg consists of three segments. The most proximal segment is called the "coxa", the second segment is called the "femur", and the final segment is the "tibia". Rotational DOFs are arranged as Euler rotations, which follow this convention:

$$\begin{pmatrix}\alpha_i \\ \beta_i \\ \gamma_i\end{pmatrix} = \begin{pmatrix}\text{rotation about body i fixed x axis} \\ \text{rotation about body i fixed y axis} \\ \text{rotation about body i fixed z axis}\end{pmatrix},$$

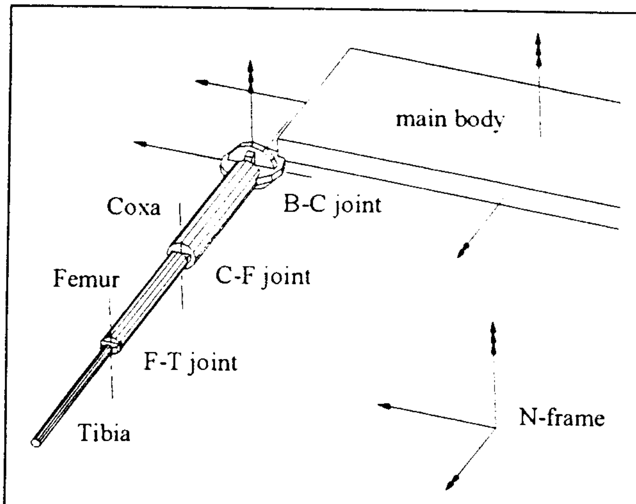

FIG. 1 SIMULATED SYSTEM IN ZEROED POSITION

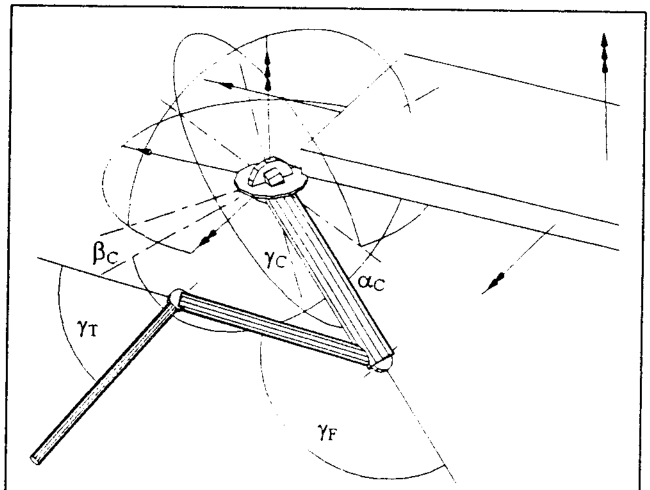

FIG. 2 SIMULATED SYSTEM IN GENERAL POSITION

$$i \in \{B\,(\text{main body}), C\,(\text{coxa}), F\,(\text{femur}), T\,(\text{tibia})\}.$$

The use of Euler angles does include possible singularities. If it is not possible to design the system representation to avoid these singular configurations, Euler parameters may be used instead.

The orientation of the main body is defined by a (3-2-1), or (γ_B-β_B-α_B) ordered set of these Euler rotations. By happenstance, the joint between the body and the coxa, called the B-C joint, allows for three rotational DOFs which occur in the same order as those used to define the orientation of the main body. The joint between the coxa and femur (C-F joint) is considered to have a single rotational DOF about an axis that is perpendicular to the plane of the leg, which is the femur z axis. The F-T joint is also considered to have a single rotation, which is about the tibia z axis.

Now it is possible to present the final D matrix, which would be inserted into Eq.(48). Singularities occur at $\beta_B = \pm\pi/2$ and $\beta_C = \pm\pi/2$. The first set represent configurations when the cockroach is pointing either straight up or straight down relative to the N-frame. This is unlikely for the scope of modeling to be done. The second set represents a B-C joint configuration which can be considered very rare for normal cockroach locomotion. In fact, this configuration can be avoided when processing actual cockroach locomotion data. The $\dot{D}$ matrix can also be derived directly from D:

$$D = \left[\begin{array}{ccc|ccc|c|c} 1 & 0 & -s\beta_B & & & & & \\ 0 & c\alpha_B & c\beta_B s\alpha_B & & 0 & & 0 & 0 \\ 0 & -s\alpha_B & c\beta_B c\alpha_B & & & & & \\ \hline & & & 1 & 0 & -s\beta_C & & \\ & 0 & & 0 & c\alpha_C & c\beta_C s\alpha_C & 0 & 0 \\ & & & 0 & -s\alpha_C & c\beta_C c\alpha_C & & \\ \hline & & & & & & 0 & \\ & 0 & & & 0 & & 0 & 0 \\ & & & & & & 1 & \\ \hline & & & & & & & 0 \\ & 0 & & & 0 & & 0 & 0 \\ & & & & & & & 1 \end{array}\right]. \quad (51)$$

It is possible to define regions in the inertia matrix of Eq.(48), and their dynamic significance:

$$\begin{array}{c} R_x \; R_y \; R_z \; \alpha_B \; \beta_B \; \gamma_B \; \alpha_C \; \beta_C \; \gamma_C \; \gamma_F \; \gamma_T \\ \left[\begin{array}{c|c|c} A & B & C \\ \hline B^T & D & E \\ \hline C^T & E^T & F \end{array}\right] \end{array}, \quad (52)$$

$A \equiv$ translational dynamics of the main body,
$B \equiv$ coupled translational and rotational dynamics of main body,
$C \equiv$ leg coupling with body translation,
$D \equiv$ rotational dynamics of main body,
$E \equiv$ leg coupling on body rotation,
$F \equiv$ rotational dynamics of individual leg.

Now our desire is to define the EOMs for a tree-like structure, which, in this case, means a system in which several multibody "appendages" extend from a common central body. In order to model a cockroach, we would have six legs attached to a central body. Eq.(48) provides the EOMs for one leg and can be used to construct iteratively the EOMs for the full body. This can be demonstrated by considering the full body inertia matrix, which can be expressed as

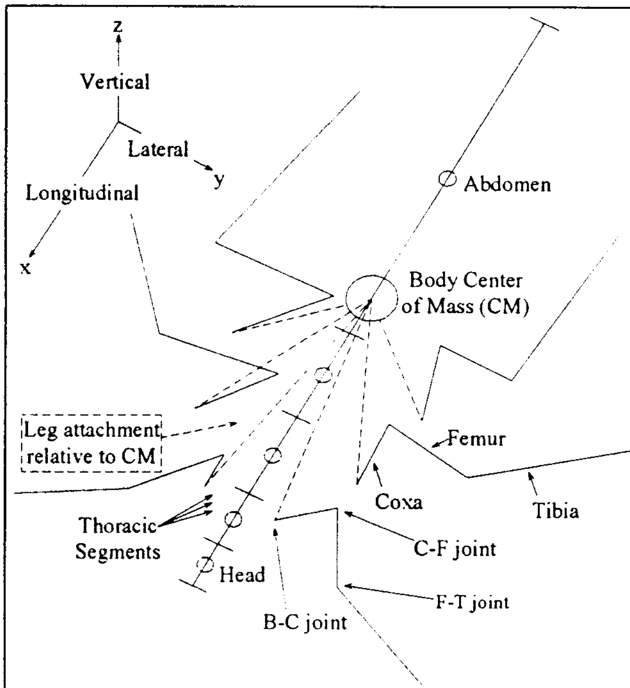

FIG. 3 OVERALL SIMULATED COCKROACH GEOMETRY AND DEFINITIONS

$$\left[\begin{array}{cccccccc} \Sigma A'_j & \Sigma B'_j & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ & \Sigma D'_j & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \\ & & F_1 & 0 & 0 & 0 & 0 & 0 \\ & & & F_2 & 0 & 0 & 0 & 0 \\ & & & & F_3 & 0 & 0 & 0 \\ & & & & & F_4 & 0 & 0 \\ & & & & & & F_5 & 0 \\ \text{sym} & & & & & & & F_6 \end{array}\right], \quad (53)$$

where j = 1,2,...6,

A'_j, B'_j, D'_j $\equiv$ same as above but using $m'_{body} = \frac{1}{6}(m_{body})$.

Therefore, as the full body EOMs are constructed, the entire mass of the body (m_{body}) will be accounted for as the leg effects are summed. The body mass does not appear in the terms for the individual legs, C_j, E_j, and F_j. Given this, it is possible to propose an algorithm that first develops Eq.(48) for each leg, and inserts regions of the leg inertia matrix into the appropriate regions in the full body inertia matrix, with summation taking place within the main body terms only, as shown in Eq.(53). This type of algorithm can also be employed when constructing the other portions of the full body EOMs. Figure 3 depicts a vector based representation of the *Blaberus* cockroach that was modeled.

NUMERICAL RESULTS

Background

Because of the primary emphasis of this paper, ground interaction is not covered in detail. A force model was used to simulate the ground. Foot position was tracked and foot forces were generated based upon the foot's position (and velocity) relative to the ground vertically, and relative to the original point of contact laterally and longitudinally. This method was flexible, in that the above EOM algorithm could be used without modification. The foot forces produce generalized torques and also act directly to affect the translation of the main body. The results of using this method were excellent. Slippage, using static and kinetic coefficients of friction, could also be simulated. This created more realistic ground interaction kinetics by disallowing an infinite static friction coefficient.

All masses and lengths were taken directly from the *Blaberus discoidalis* cockroach. Inertias were derived by using rods to model femurs and tibias, and semiellipsoids to model coxas. Inertias for the main body were derived based upon the dimensions and locations of the various thoracic segments, the head, and the abdomen. High-speed video of the insect, at approximately 200 frames per second, was digitized in two views (lateral and ventral). This data was then used to reproduce the true joint angles versus time for various gaits of the animal. Therefore, the full body EOMs could be driven based upon actual insect locomotion using PD feedback control on each joint.

Preliminary Results

One of the primary results that has helped to validate the model deals with leg function. It has been shown using force plate measurements that the front, middle, and back legs of the roach each perform specific functions during locomotion (Full et al., 1991). When considering forward, or longitudinal locomotion at constant speed, the front legs serve to brake, or decelerate the body, while the back legs serve to accelerate the body. The middle legs perform a combination of both functions, initially decelerating and then accelerating. This phenomenon has been realized in the simulation.

The plots of Figure 4 indicate ground reactions for one stance interval. The lower traces in each plot show longitudinal reactions, while the upper show vertical reactions. If one considers the impulse during stance due to these longitudinal reactions, the following results are produced.

$$\int \begin{pmatrix} \text{Longitudinal Reaction} \\ \text{Front Leg} \end{pmatrix} \cong -48.1 \left(\frac{\text{cm g}}{\text{s}} \right)$$

$$\int \begin{pmatrix} \text{Longitudinal Reaction} \\ \text{Middle Leg} \end{pmatrix} \cong -0.4 \left(\frac{\text{cm g}}{\text{s}} \right)$$

$$\int \begin{pmatrix} \text{Longitudinal Reaction} \\ \text{Back Leg} \end{pmatrix} \cong 49.8 \left(\frac{\text{cm g}}{\text{s}} \right)$$

It should be noted that this result comes from actual cockroach locomotion, which accounts for much of the "noise" and other variability in the data. This preliminary result serves to verify this simulation in part.

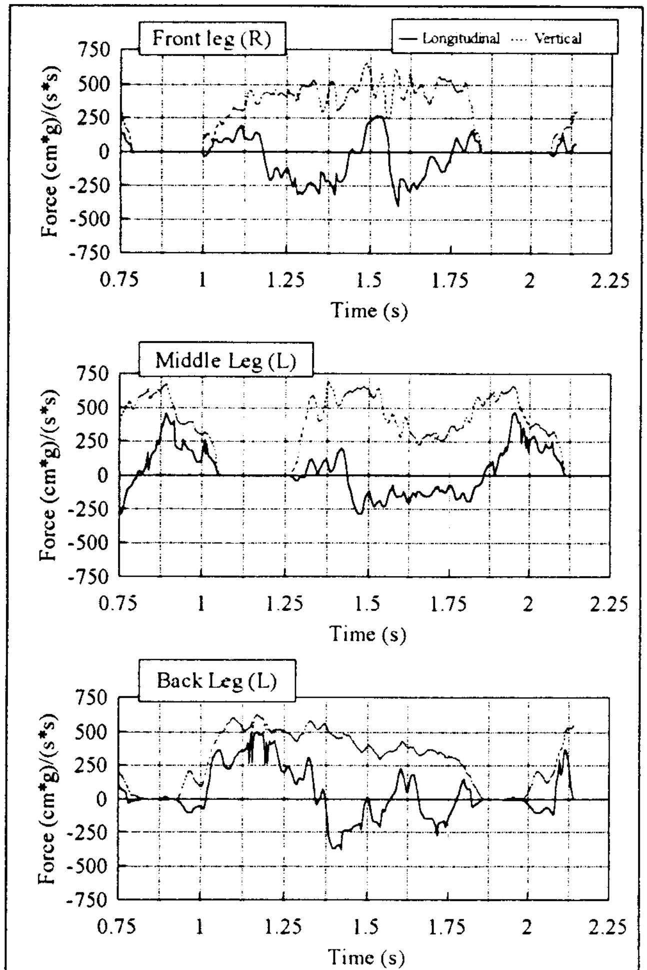

FIG. 4 LEG FUNCTION OBSERVED IN SIMULATION

Insight has also been gained into how a cockroach-like hexapod could minimize joint torques. Simulating a standing roach has revealed a well defined relationship involving joint controller stiffness, ground reactions, and joint torque effort ($\underline{\tau}_G^T \underline{\tau}_G$). In the approach described in this paper, a controller with infinite stiffness would drive joint angle error to zero and produce purely vertical ground reactions at the feet. This results in large joint torque effort in each leg. By lowering controller stiffnesses, the joint torque efforts drop dramatically. At the same time significant lateral and longitudinal ground reactions result, which tend towards the main body. Further relaxation of the controller stiffnesses will actually increase joint torque efforts. Therefore a minimum state exists. The data in Table 1 demonstrates this for the front legs.

CONCLUSIONS

A general formulation for open-chain multibody structures has been applied to the locomotion of a cockroach. The formulation

TABLE 1 TORQUE EFFORT IN STANCE WITH VARIOUS CONTROLLERS

Average Position Error (deg)	Torque Effort
0.001	41.7
0.015	27.5
0.084	21.0
0.230	21.6
0.525	22.7
0.962	25.1

is compact and easily implemented, providing an efficient means of constructing the equations of motion for any general tree-like multibody structure. The formulation method is especially well suited for computer coding. As an example, a simulation was written using this method which models the locomotion of a *Blaberus discoidalis* cockroach. A force model was used to simulate contact with the ground, which included static and kinetic frictional effects. Joint angles are acquired from high-speed video of the roach, and output from the simulation includes body motion, joint torques, and ground reaction forces. Preliminary numerical results have agreed with experimental observation of the animal.

This formulation could be used to model all types of pedal systems, such as bipeds, quadrupeds, etc. It is also well suited for manipulators using revolute joints, or, by employing some form of discretization, flexible structures like snakes. One goal of this research is to use this simulation in aiding the design and construction of a cockroach-like robot.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. J.Watson for providing data of digitized cockroach locomotion. G.M.Nelson is supported by grant NGT-51194 through the NASA Marshall Space Flight Center. Additional support was provided by the Office of Naval Research, grant N00014-90-J-1545.

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